**GREENWOOD COLLEGE**

**Mathematics Methods Units 3 & 4**

**Test 7 Interval Estimates for Proportions 2019**

Name Mark **/18**

**All electronic devices must be switched off and in bags.**

**Access to Formulae Sheet allowed. No notes.**

**No calculators allowed in this section. Time limit 20 minutes.**

1. [2,2 = 4 marks]

**a)** Under what two circumstances can you assume that the distribution of sample proportions of a distribution is normally distributed?

*The original distribution is normally distributed* ✓

*The quantity of sample means is greater than 30* ✓

**b)** If has mean and standard deviation  **,** what are the mean and standard deviations of the distribution of 64 sample proportions?

The mean will be around the same i.e. 27 ✓

Standard deviation = = ✓

2. [1,3,3 = 7 marks]

The amount of granulated coffee dispensed to make a cup of coffee is uniformly distributed from 4.5g to 5.5g.

**a)** State the probability that in any cup of coffee produced by this coffee machine that the mass of granulated coffee used is more than 5.4g

or 0.1 ✓

The machine was used 36 times and the proportion of times more than 5.4 g of granulated coffee was dispensed was recorded. This was repeated 100 times so that a collection of 100 sample proportions was obtained.

**b)** Describe the sampling distribution of sample proportions of size 36 for the mass of granulated coffee dispensed exceeding 5.4g, stating its mean and standard deviation.

*As sample size n = 36 (>30)*

*Sample proportions are normally distributed* ✓

*Mean =* ✓

*Standard deviation = =* ✓

**c)** Describe the frequency distribution of the 100 sample proportions of the mass of granulated coffee used exceeding 5.4g, stating its mean and standard deviation.

*As the number of samples N is large the frequency distribution tends towards its sampling distribution, hence it is approximately normal* ✓

*Mean =* ✓

*Standard deviation = =* ✓

3. [3,4 = 7 marks]

A student planned to investigate what proportion of the 1260 students at their school had access to more than one computer at home.

**a)** The student thought of the following three ways to select a sample from the population. Briefly discuss the main source of bias in each method.

**i)** Wait at the bus-bay after school and ask the first 50 students who show up.

(1 mark)

|  |
| --- |
| **Solution** |
| Biased towards students who catch bus. |
| **Specific behaviours** |
| ✓ identifies group bias |

**ii)** Advertise the survey in a whole school assembly and ask the first 50 students who volunteer to stay behind. (1 mark)

|  |
| --- |
| **Solution** |
| Self-selected samples are likely to suffer from non-response bias. |
| **Specific behaviours** |
| ✓ identifies self-selection bias |

**iii)** Select and ask every 100th student from the school roll. (1 mark)

|  |
| --- |
| **Solution** |
| Small samples likely to be biased - in this case sample of only 13. |
| **Specific behaviours** |
| ✓ identifies small sample bias |

**b)** Assuming that 80% of students had access to more than one computer at home, the student carried out 100 simulations in which a sample proportion was calculated from a random sample of 64 students.

(i) Explain why it is reasonable to expect that the distribution of the sample proportions would approximate normality. (2 marks)

|  |
| --- |
| **Solution** |
| The sample size of 64 is reasonably large ().  Also, both and exceed the rule-of thumb minimum of 10. |
| **Specific behaviours** |
| ✓ states large sample size  ✓ indicates dependence on both and |

(ii) Determine the mean and standard deviation of the normal distribution that the sample proportions would approximate. (2 marks)

|  |
| --- |
| **Solution** |
| Mean of  Standard deviation of |
| **Specific behaviours** |
| ✓ states mean  ✓ states standard deviation |

END OF SECTION

**GREENWOOD COLLEGE**

**Mathematics Methods Units 3 & 4**

**Test 7 Interval Estimates for Proportions 2019**

Name Mark **/36**

**All electronic devices must be switched off and in bags.**

**Access to Formulae Sheet and one sheet of A4 notes allowed. Use of approved calculators is assumed in this section.**

**Time limit 35 minutes.**

**4.** **[ 5,1,2,2,3 = 13 marks]**

The fat content (in grams) of 30 randomly selected pasties at a local Greenwood bakery was recorded:

15.1 14.8 13.7 15.6 15.1 16.1 16.6 17.4 16.1 13.9

17.5 15.7 16.2 16.6 15.1 12.9 17.4 16.5 13.2 14.0

17.2 17.3 16.1 16.5 16.7 16.8 17.2 17.6 17.3 14.8

**a)** Determine a 90% confidence interval for the mean fat content of all pasties made at this bakery.

*Using technology* ✓

✓

✓

*So confidence interval* = ✓

✓

**b)** Make a summary statement of your findings in part **a)**.

*We are 90% confident that the mean fat content of all pasties produced lies between 15.7912g and 16.0088g* ✓

**c)** What would be the equivalent 95% confidence interval in this situation?

✓✓

**d)** Compare the margins of error in both instances.

Error for *90% confidence =0.1088*

Error for *95% confidence = 0.1296* ✓

Margin of error is smaller in =the 90% confidence interval ✓

**e)** What sample size would be required to maintain a margin of error of 0.3g with a 95% confidence interval?

Solve 0.3 = 1.96 x ✓ ✓ so 80 ✓

**5. [ 3,1 = 4 marks]**

A regular pentagon spinner has sectors numbered 1, 1, 2, 3 , 4 . Estimate the probability that when the pentagon is spun 400 times, the result of 1 occurs:

**a)** more than 37.5% of the time

) or ) mean ✓

✓

= 0.8463 (0.8338 by ) ✓

**b)** less than 43.75% of the time

= 0.9371 (0.9301 by ) ✓

**6. [ 4,3,3 = 10 marks]**

An unbiased six-sided die is rolled 75 times. This is repeated 200 times to form 200 samples each consisting of 75 rolls of the die. Event S is defined as the roll of the die that produces a six.

**a)** Calculate the probability that a randomly chosen sample has a sample proportion of event S that exceeds 15%.

or normal distribution ✓

mean ✓

✓

= 0.6508 ✓

**b)** Estimate with reasons the expected number of samples with sample proportions of event S that exceeds 15%.

✓ 0.6508 x 200 = 130.16 ✓ so ✓

**c)** In a separate experiment the same die was rolled times. Determine if the standard deviation of the sampling distribution of this event is not to exceed 0.04.

✓

✓

So ✓

**7. [ 5,2,2 = 9 marks]**

The management at a conference centre was concerned about the quality of the free pens that it provided in its meeting rooms. A staff member tested a random sample of 150 pens and found that 18 of them fail to write.

**a)** If is the true proportion of pens that fail to write and is the corresponding sample proportion, use the above sample to determine

**i)** .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates |

**ii)** the approximate margin of error for a 98% confidence interval for .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ calculates z-score  ✓ calculates standard error  ✓ calculates margin of error |

**iii)** an approximate 98% confidence interval for .

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ evaluates interval |

**b)** The stationery company that supplies pens to the conference centre claim that no more than 3 in 50 pens fail to write. Use your previous working to comment on the validity of this claim. (2 marks)

|  |
| --- |
| **Solution** |
| .  The interval calculated in (a) contains 0.06 and so the claim is valid. |
| **Specific behaviours** |
| ✓ compares proportion to confidence interval.  ✓ states claim is valid |

**c)** Comment on how the margin of error would change in (a) (ii) if

**i)** the quality of the pens had been better. (1 mark)

|  |
| --- |
| **Solution** |
| Decrease, as is further from 0.5. |
| **Specific behaviours** |
| ✓ states change |

**ii)** the required level of confidence decreased. (1 mark)

|  |
| --- |
| **Solution** |
| Decrease, as z-score lower. |
| **Specific behaviours** |
| ✓ states change |

END OF PAPER